PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 4369–4372

International Journal of
HEAT and MASS
TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

On the ratio of heat to mass transfer coefficient for water evaporation and its impact upon drying modeling

Xiao Dong Chen [a,*], Sean X.Q. Lin [a], Guohua Chen [b]

[a] *Department of Chemical and Materials Engineering, The University of Auckland, Private Bag 92019, Auckland City, New Zealand*
[b] *Department of Chemical Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China*

Received 19 November 2001; received in revised form 14 March 2002

1. Introduction

Liquid evaporation and drying are common processes for concentrating liquids and for making solid products. The interfacial conditions must be expressed in correct mathematical terms so that both evaporation and drying can be modeled adequately. In many circumstances, the conventional heat and mass transfer analogy, e.g. the Colburn–Chilton analogy, is employed to yield the corresponding mass transfer coefficient once the heat transfer coefficient is known [1–4]. It has not been mentioned though in literature, since the wet-bulb temperature has been accepted as a property of the environment that water evaporates or drying proceeds in. A steady-state heat balance is attained for thin sample liquid layer at this temperature. This heat balance in fact yields a relationship of h/h_m, which can be shown to be different from that calculated using the conventional heat and mass transfer analogy. In this technical note, the above has been demonstrated and the potential impact of this is illustrated in simulations of drying of the thin skim milk layer with known physical properties.

2. The heat-mass transfer coupling under small *Biot* number conditions for pure water evaporation—leading to a different formulation of h/h_m

When *Biot* number is less than 0.1, the temperature of the sample liquid layer is considered to be uniform throughout. As evaporation proceeds, the sample temperature becomes equal to the wet-bulb temperature, corresponding to the ambient conditions [5,6]. Provided sufficient water content, this temperature could be maintained for some time. As such the steady-state heat balance for a water droplet can be written as:

$$hA(T_\infty - T_{\mathrm{wb}}) = \Delta H_{\mathrm{L}} h_{\mathrm{m}} A(\rho_{\mathrm{v,sat}}(T_{\mathrm{wb}}) - \rho_{\mathrm{v},\infty}) \tag{1}$$

On the LHS of the above equation, heat transfer rate due to convection is presented and on the RHS the heat dissipation rate due to evaporation is presented. Based on this heat balance, the ratio of heat transfer coefficient (h) to mass transfer coefficient (h_m) is the following:

$$\frac{h}{h_{\mathrm{m}}} = \frac{(\rho_{\mathrm{v,sat}}(T_{\mathrm{wb}}) - \rho_{\mathrm{v},\infty})\Delta H_{\mathrm{L}}(T_{\mathrm{wb}})}{(T_\infty - T_{\mathrm{wb}})} = \phi(\rho_{\mathrm{v},\infty}, T_{\mathrm{wb}}, T_\infty) \tag{2}$$

Eq. (2) is of course very different from that of the conventional heat and mass transfer analogy, e.g. the Colburn–Chilton analogy [1–4]:

$$\frac{h}{h_{\mathrm{m}}} = \frac{k_{\mathrm{v,f}}}{D_{\mathrm{v,f}}} Le^{-n} \tag{3}$$

where ‘f’ represents the film value, ‘v’ represents water vapor. The power ‘n’ is often approximated as 1/3. The *Lewis* number (Le) is the ratio of the thermal and mass diffusivities:

$$Le = \frac{\alpha_{\mathrm{v,f}}}{D_{\mathrm{v,f}}} \tag{4}$$

Two examples of water evaporation are given in Fig. 1. Two ambient temperatures are considered, i.e. 40 and 70 °C respectively. One can see that the two Eqs. (2) and

---

[*] Corresponding author. Tel.: +64-9-373-7599x7004; fax: +64-9-373-7463.
E-mail address: d.chen@auckland.ac.nz (X.D. Chen).

PII: S0017-9310(02)00139-4

Nomenclature

ΔH_L	latent heat of water vaporization ($\mathrm{J\,kg^{-1}}$)
A	surface area ($\mathrm{m^2}$)
b, c, d	constants in Eq. (20)
C, m, n	constants in Eq. (9)
C_p	specific heat capacity ($\mathrm{J\,kg^{-1}\,K^{-1}}$)
D	diffusivity ($\mathrm{m^2\,s^{-1}}$)
h	heat transfer coefficient ($\mathrm{W\,m^{-2}\,K^{-1}}$)
h_m	mass transfer coefficient ($\mathrm{m\,s^{-1}}$)
k	thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)
L	characteristic length (m)
Le	Lewis number
m	mass (kg)
Nu	Nussult number
Re	Reynolds number
RH	relative humidity
Sc	Schmidt number
Sh	Sherwood number
T	temperature (°C)
t	time (s)
X	water content on dry basis ($\mathrm{kg\,kg^{-1}}$)

Greek symbols

ψ	new dimensionless parameter (Eq. (12))
ρ	concentration ($\mathrm{kg\,m^{-3}}$)
ϕ	ratio in Eq. (2)
α	thermal diffusivity ($\mathrm{m^2\,s^{-1}}$)

Subscripts

∞	bulk or ambient
f	film coefficient
L	latent heat
m	mass transfer
s	surface or solids
sat	saturation
v	vapor
w	water
wb	wet-bulb

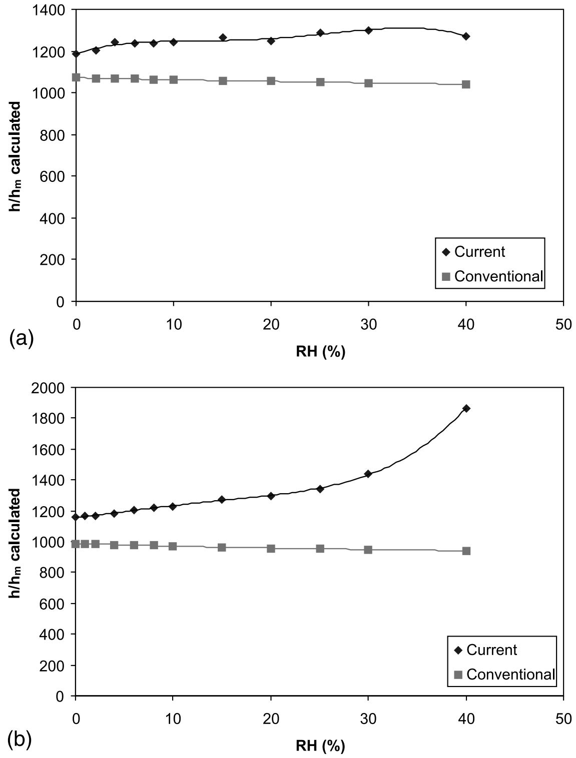

Fig. 1. Comparison between the current model predictions of h/h_m for pure water and that of the conventional model for different ambient humidities: (a) 40 °C and (b) 70 °C.

(3) give considerably different results. Considering the following well-known dimensionless parameters:

$$Nu = \frac{hL_c}{k_{v,f}}, \qquad Sh = \frac{h_m L_c}{D_{v,f}} \tag{5}$$

One can write the ratio of the two dimensionless parameters as below:

$$\frac{Nu}{Sh} = \frac{h}{h_m}\frac{D_{v,f}}{k_{v,f}} \tag{6}$$

Based on Eq. (2) one can have

$$\frac{Nu}{Sh} = \phi\frac{D_{v,f}}{k_{v,f}} \tag{7}$$

If Nu correlation is expressed in one of the common forms, e.g.

$$Nu = C\,Re^m\,Pr^n \tag{8}$$

The corresponding Sh correlation can be written as

$$Sh = \frac{C}{\phi}\left(\frac{k_{v,f}}{D_{v,f}}\right)Le^{-n}\,Re^m\,Sc^n \tag{9}$$

by combining Eqs. (7) and (8), where the Schmidt number is $Sc = \alpha_{v,f}/D_{v,f}$.

A new dimensionless parameter may be introduced after modifying Eq. (9), as follows:

$$Sh = C\left(\frac{k_{v,f}}{D_{v,f}}\right)\left(\frac{1}{\rho_{v,f}C_{p_{v,f}}}\right)\frac{\rho_{v,f}C_{p_{v,f}}(T_\infty - T_{wb})}{\Delta H_L(T)(\rho_{v,sat}(T_{wb}) - \rho_{v,\infty})} \times Le^{-n}Re^m Sc^n \quad \text{or}$$

$$Sh = C Le^{1-n}\psi Re^m Sc^n \tag{10}$$

where, the new dimensionless parameter is defined

$$\psi = \frac{\rho_{\mathrm{v,f}} C_{p_{\mathrm{v,f}}} (T_\infty - T_{\mathrm{wb}})}{\Delta H_{\mathrm{L}}(T)(\rho_{\mathrm{v,sat}}(T_{\mathrm{wb}}) - \rho_{\mathrm{v},\infty})} \tag{11}$$

The new dimensionless group ψ is a measure of the ratio of the sensible heat gain by the vapor to the heat due to phase change. Correspondingly, Eq. (2) can be re-written as

$$\frac{h}{h_{\mathrm{m}}} = \rho_{\mathrm{v,f}} C_{p_{\mathrm{v,f}}} \psi^{-1} \tag{12}$$

Since we now have a different form of h/h_{m}, i.e. Eq. (12), we should expect that the predicted drying behavior of a moist material would be different when using model Eqs. (2) and (3) respectively.

3. Drying of moist materials under heat and mass transfer 'thin' conditions (*Bi* < 0.1)

For very small dimensions of a moist material and when the convective heat and mass transfer around the materials are made small, both the *Biot* number for heat transfer and the *Biot* number for mass transfer can be made smaller than 0.1 [1]. This means that the non-uniformity of the temperature distribution and the non-uniformity of the water content in the materials can be neglected. Under these circumstances, the following transient mass (water) and the energy balances of moist porous media in a convective environment can be written.

Mass balance

$$m_{\mathrm{s}} \frac{\mathrm{d}X}{\mathrm{d}t} = -h_{\mathrm{m}} A(\rho_{\mathrm{v,sat}}(T)\mathrm{RH_s} - \rho_{\mathrm{v},\infty}) \tag{13}$$

Heat balance

$$mC_p \frac{\mathrm{d}T}{\mathrm{d}t} = hA(T_\infty - T) - \Delta H_{\mathrm{L}} m_{\mathrm{s}} \left| \frac{\mathrm{d}X}{\mathrm{d}t} \right| \tag{14}$$

Eq. (14) can be rewritten as

$$(1+X)m_{\mathrm{s}} \left(\frac{1}{1+X} C_{ps} + \frac{X}{1+X} C_{pw} \right) \frac{\mathrm{d}T}{\mathrm{d}t} = hA(T_\infty - T) - \Delta H_{\mathrm{L}} m_{\mathrm{s}} \left| \frac{\mathrm{d}X}{\mathrm{d}t} \right| \tag{15}$$

Eq. (15) can be further re-written as

$$m_{\mathrm{s}}(C_{ps} + XC_{pw}) \frac{\mathrm{d}T}{\mathrm{d}t} = hA(T_\infty - T) - \Delta H_{\mathrm{L}}(T) m_{\mathrm{s}} \left| \frac{\mathrm{d}X}{\mathrm{d}t} \right| \tag{16}$$

Dividing Eq. (16) by Eq. (13) gives

$$(C_{ps} + XC_{pw}) \frac{\mathrm{d}T}{\mathrm{d}X} = -\left(\frac{h}{h_{\mathrm{m}}} \right) \frac{(T_\infty - T)}{(\rho_{\mathrm{v,sat}}(T)\mathrm{RH_s} - \rho_{\mathrm{v},\infty})} + \Delta H_{\mathrm{L}}(T) \tag{17}$$

Based on the earlier analysis, h/h_{m} could be calculated using either Eq. (2) or (3), two different results are expected from Eq. (17).

Chilton–Colburn analogy

$$(C_{ps} + XC_{pw}) \frac{\mathrm{d}T}{\mathrm{d}X} = -\left(\frac{k_{\mathrm{f}}}{D_{\mathrm{f}}} \right) Le^{-n} \frac{(T_\infty - T)}{(\rho_{\mathrm{v,sat}}(T)\mathrm{RH_s} - \rho_{\mathrm{v},\infty})} + \Delta H_{\mathrm{L}}(T) \tag{18}$$

The heat balance approach outlined in this note

$$\begin{aligned}(C_{ps} + XC_{pw}) \frac{\mathrm{d}T}{\mathrm{d}X} &= -\phi \frac{(T_\infty - T)}{(\rho_{\mathrm{v,sat}}(T)\mathrm{RH_s} - \rho_{\mathrm{v},\infty})} + \Delta H_{\mathrm{L}}(T) \\ &= -\left[\frac{\rho_{\mathrm{v,sat}}(T_{\mathrm{wb}}) - \rho_{\mathrm{v},\infty}}{\rho_{\mathrm{v,sat}}(T)\mathrm{RH_s} - \rho_{\mathrm{v},\infty}} \right] \left(\frac{T_\infty - T}{T_\infty - T_{\mathrm{wb}}} \right) \\ &\quad \times \Delta H_{\mathrm{L}}(T_{\mathrm{wb}}) + \Delta H_{\mathrm{L}}(T)\end{aligned} \tag{19}$$

Both equations can predict a minimum value of temperature, corresponding to the 'initial cool-down' process if the initial temperature of the moist material is higher than the wet-bulb temperature. To show quantitatively the kind of differences in drying predictions between the two approaches (Eqs. (2) and (3)), the thin layer drying process for skim milk has been simulated. For simplicity, we firstly assume that extremely dry gas is used as the drying medium, i.e. $\rho_{\mathrm{v},\infty} \approx 0$. Taking the following equilibrium isotherm function [7] to approximate the relationship between water content and relative humidity at any temperature:

$$\mathrm{RH_s} = \exp\left(-b \frac{X^{-c}}{T^d} \right) \tag{20}$$

This equation can be incorporated in Eqs. (18) and (19). For skim milk, b, c and d are taken as 9.5665×10^{14}, 2.18 and 15 respectively [7]. Skim milk is a widely examined food fluid [8–13]. The physical properties of the skim milk are taken from Sano and Keey [9]. The physical properties of the water vapor and air are taken from Incropera and De Wit [1]. Eqs. (18) and (19) can both be integrated from the initial water content, i.e. $X_0 = 2.333$ (kg kg^{-1}) and $T = T_{\mathrm{wb}}$, to yield a relationship between T and X under different sets of ambient conditions. Fig. 2(a) shows comparison between the predictions made using the conventional approach (Eq. (18)) and that using the heat balance approach (Eq. (19)). The initial solids content is 30 wt.%. The flat sections of the curves predicted by the model correspond to $\mathrm{RH_s} \approx 100\%$. Qualitatively, the two approaches give similar trends. The main differences between the two model predictions are at the high water content end. The heat balance approach gives the exact results of the wet-bulb temperatures.

Bearing in mind the sort of differences in T–X relationships shown in Fig. 2(a), it is possible that the weight-loss curves under the same drying conditions would be different in between the predictions using the

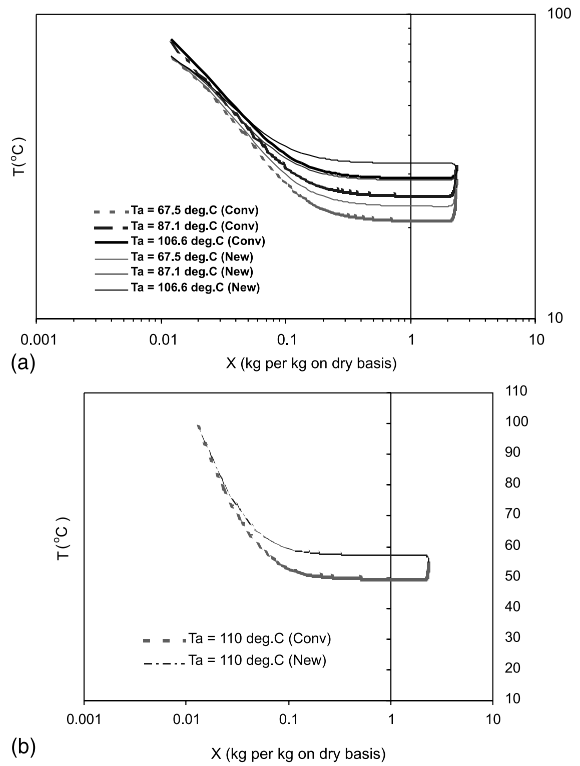

Fig. 2. (a) A comparison between the heat balance model predictions and the conventional model predictions for the zero humidity case at $Bi \leqslant 0.1$ (skim milk layer; air velocity = 0.45 m s^{-1}, 30 wt.% initial solids, air humidity = 0.0001 g g^{-1}). (b) A comparison between the heat balance model predictions and the conventional model predictions for the non-zero humidity case at $Bi \leqslant 0.1$ (skim milk layer). Initial droplet temperature $T_0 = 55$ °C, wet-bulb temperature $T_{wb} = 57.3$ °C, ambient vapor concentration $\rho_{v,\infty} = 0.0579$ kg m^{-3} (RH$_\infty \approx 7\%$ at T_a), ambient temperature $T_a = 110$ °C.

two approaches. Fig. 2(b) shows an even more significant difference between the two models. The conditions for this figure are: initial droplet temperature $T_0 = 55$ °C, wet-bulb temperature $T_{wb} = 57.3$ °C, ambient vapor concentration $\rho_{v,\infty} = 0.0579$ kg m^{-3} (RH$_\infty \approx 7\%$ at T_a), ambient temperature $T_a = 110$ °C. These conditions lead to a $\phi = 2541.5$. The heat balance approach (Eq. (19)) predicts an initial 'warm-up' period but the conventional model predicts a 'cool-down' period.

4. Conclusions

This note has addressed one interesting aspect of evaporation and drying modeling. The heat to mass transfer coefficient ratio, obtained based on the heat balance equation for a thin water layer, is different from that derived from the conventional heat and mass transfer analogy. Here the Colburn–Chilton analogy was used. Based on the simulations carried out for drying of the thin skim milk layer, considerable differences in temperature and water content relationships of the liquid layer have been found when using the two different approaches. This has raised an important fundamental question for future drying modeling work.

References

[1] F.P. Incropera, D.P. De Wit, Fundamentals of Heat and Mass Transfer, third ed., Wiley, New York, 1990, pp. 360–364.
[2] A. Bejan, Heat Transfer, John Wiley & Sons, New York, 1993, pp. 597–602.
[3] J.P. Holman, Heat Transfer, fourth ed., McGraw-Hill, Tokyo, 1976, pp. 434–435.
[4] E.L. Cussler, Diffusion—Mass Transfer in Fluid Systems, Cambridge, 1984, pp. 229–232.
[5] R.B. Keey, Drying of Loose and Particulate Materials, Hemisphere Publishing Corporation, New York, 1992, pp. 223–227, University Press, Cambridge, pp. 447–449.
[6] C. Strumillo, T. Kudra, Drying: Principles, Applications and Design, Gordon and Breach Science Publishers, New York, 1986, pp. 18–77.
[7] L.V. Chong, X.D. Chen, A mathematical model of the self-heating of spray dried food powders containing fat, protein, sugar and moisture, Chem. Engng. Sci. 54 (19) (1999) 4165–4178.
[8] A.E. Wijlhuizen, P.J.A.M. Kerkhof, S. Bruin, Theoretical study of the inactivation of phosphatase during spray drying of skim-milk, Chem. Engng. Sci. 34 (1979) 651–660.
[9] Y. Sano, R.B. Keey, The drying of a spherical particle containing colloidal material into a hollow sphere, Chem. Engng. Sci. 37 (6) (1982) 881–889.
[10] G. Ferrari, G. Meerdink, P. Walstra, Drying kinetics for a single droplet of skim milk, J. Food Engng. 10 (1989) 215–230.
[11] S. Nesic, The evaporation of single droplets—experiments and modeling, in: A. Mujumdar (Ed.), Drying'89, 1989, pp. 386–393.
[12] M.J. Stevenson, X.D. Chen, A. Fletcher, The effect of fat content on the drying of milk products, in: A. Mujumdar (Ed.), Drying'98 (IDS'98), 1998, pp. B1200–1206.
[13] M.J. Stevenson, Computational Modeling of Drying of Milk, ME Thesis, Department of Chemical and Materials Engineering, The University of Auckland, 1999.